National Aeronautics and Space Administration

Exploring Space Through ALGEBRA

EDUCATOR EDITION

Algebra I

Weightless Wonder – Reduced Gravity Flight

Instructional Objectives
Students will
- analyze data to derive a solution to a real-world problem;
- solve and evaluate quadratic functions; and
- find the maximum of a quadratic function.

Prerequisites

Students should have prior experience working with the properties of quadratic equations. They should understand the different parts of a parabola and its properties as well as how to find values, such as minimums, maximums, and a vertex point, given a quadratic equation.

Background

This problem is part of a series of problems that apply Algebra principles to NASA's Vision for Space Exploration.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

The Vision for Space Exploration includes returning the space shuttle safely to flight, completing the International Space Station, developing a new exploration vehicle and all the systems needed for embarking on extended missions to the Moon, Mars, and beyond.

In our quest to explore, humans will have to adapt to functioning in a variety of gravitational environments. Earth, Moon, Mars and space all have different gravitational characteristics. Earth's gravitational force is referred to as one Earth gravity, or 1g. Since the Moon has less mass than the Earth, its gravitational force is only one sixth that of Earth, or 0.17g. The gravitational force on Mars is equivalent to about 38% of Earth's gravity, or 0.38g. The gravitational force in space is called microgravity and is very close to zero-g.

Grade Level
8-12

Subject Area
Mathematics: Algebra I

Key Concept
Evaluating quadratic functions

Teacher Prep Time
5 minutes

Problem Duration
15-30 minutes

Technology
Calculator

Materials
- Student Edition
- Graph Paper
- Markers

Degree of Difficulty
Moderate to Difficult

Skill
Evaluate quadratic functions; find maximums; recognize y-intercepts; connect equations and graphs to a real-world problem.

NCTM Principles and Standards
- Number and Operations
- Algebra
- Problem Solving
- Communication
- Connections
- Representation

Figure 1: C-9 jet going into a parabolic maneuver.

Figure 2: Astronaut crew training onboard the C-9 aircraft in preparation for the Microgravity Science Laboratory missions flown on the Space Shuttle Columbia in April and July of 1997.

When astronauts are in orbit, either in the space shuttle or on the International Space Station (ISS), Earth's gravitational force is still working on them. However, astronauts maintain a feeling of weightlessness, since both the vehicle and crew members are in a constant state of free-fall. Even though they are falling towards the Earth, they are traveling fast enough around the Earth to stay in orbit. During orbit, the gravitational force on the astronauts relative to the vehicle is close to zero-g.

The C-9 jet is one of the tools utilized by NASA to simulate the gravity, or reduced gravity, astronauts feel once they leave Earth (Figure 1). The C-9 jet flies a special parabolic pattern that creates several brief periods of reduced gravity. A typical NASA C-9 flight goes out over the Gulf of Mexico, lasts about two hours, and completes between 40 and 60 parabolas. These reduced gravity flights are performed so astronauts, as well as researchers and their experiments, can experience the gravitational forces of the Moon and Mars and the microgravity of space.

By using the C-9 jet as a reduced gravity research laboratory, astronauts can simulate different stages of spaceflight. This can allow crew members to practice what might occur during a real mission. These reduced gravity flights provide the capability for the development and verification of space hardware, scientific experiments, and other types of research (Figure 2). NASA scientists can also use these flights for crew training, including exercising in reduced gravity, administering medical care, performing experiments, and many other aspects of spaceflight that will be necessary for an exploration mission. A flight on the C-9 jet is the next best thing to blasting into orbit!

For more information on NASA's Weightless Wonder and reduced gravity research, see the 13 minute video at http://microgravityuniversity.jsc.nasa.gov/video/RGSFOP_video.mpg. For more information about the Vision for Space Exploration, visit www.nasa.gov.

NCTM Principles and Standards

Number and Operations
- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- Judge the reasonableness of numerical computations and their results.

Algebra
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency – mentally or with paper and pencil in simple cases and using technology in all cases.
- Draw reasonable conclusions about a situation being modeled.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Communication
- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections
- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation
- Recognize and use connections among mathematical ideas.
- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.

Problem

To prepare for an upcoming mission, an astronaut participated in a C-9 flight simulating microgravity, or close to zero-g. The pilot flew out over the Gulf of Mexico, dove down to increase to a maximum speed then climbed up until the nose was at a 45° angle with the ground. To go into a parabolic maneuver, the pilot then cut the thrust of the engine letting the nose of the plane continue to rise then come back down at a - 45° angle with the ground. Ending the maneuver, the pilot throttled the engine back up and began another dive to prepare for the next parabola. The pilot completed 50 parabolas during the 2 hour flight.

Figure 3 shows the movement of the plane during a typical flight. The parabolic maneuver, where microgravity is felt, is highlighted. This is the part of the flight that you will focus on for the following questions.

The function $y = -4.9t^2 + 87.21t + 9144$ describes the altitude (y), in meters (m), of the plane in relation to the time (t), in seconds (s), after it started the parabolic maneuver. You will use this function to analyze the parabolic flight of the C-9.

Students will use a quadratic function to analyze the parabolic flight path of the C-9 jet. This information will help students understand how reduced gravity can be experienced on Earth.
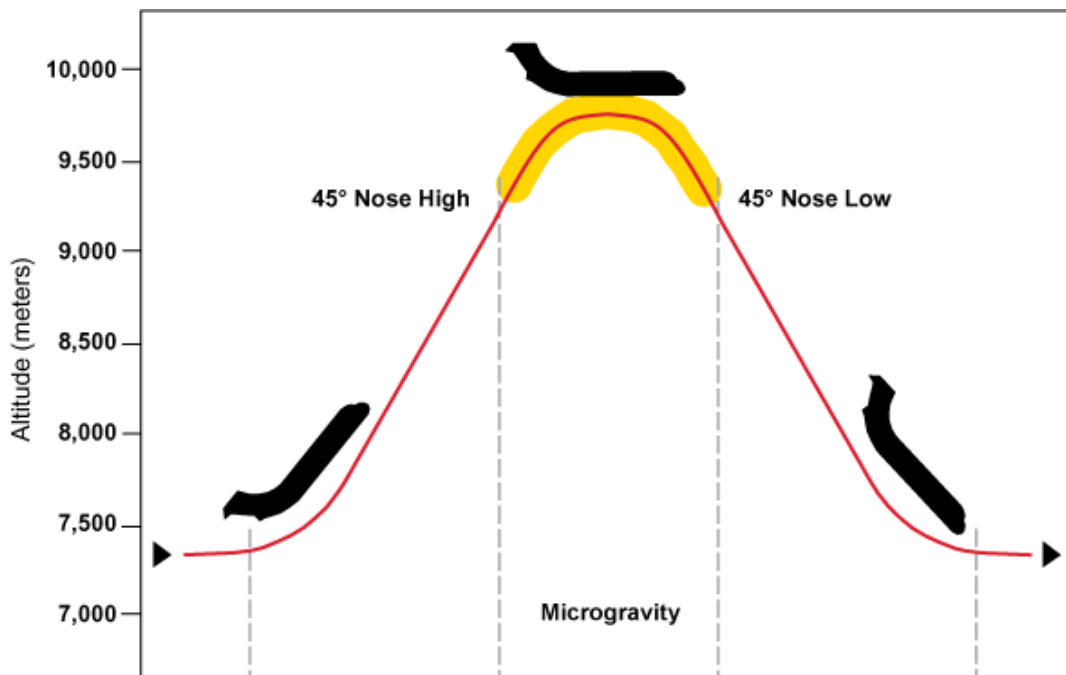

Figure 3: A typical microgravity maneuver.

Lesson Development

Students should work in groups of three or four. They may solve using graph paper and/or a graphing calculator. They should discuss different strategies for solving the problems. Ask students to record their answers with solutions on graph paper and post them around the classroom.

Wrap-Up

Have students partner with members of different groups. Ask the pairs to rotate around the classroom observing all of the solutions to the problems that are posted. The pair should then discuss and decide which is the best strategy. Then, as a class discussion, ask the students to discuss the various approaches and choose what they believe to be the best strategy to solve the problems. Students should be able to explain their reasoning.

Extensions

Students may complete the activity, *Weightless Wonder – Reduced Gravity Flight: Part II* using TI graphing calculators to evaluate quadratic functions. This helps students learn how to graph quadratic equations, find maximums, and evaluate values using the TI graphing calculator.

Solution Key (One approach)

1. Using the defined function, at what altitude did the astronaut start to feel microgravity?

   At $t = 0$, which would be 9,144 meters.

2. Calculate the length of time the astronaut experienced microgravity during one parabolic maneuver.

   Substitute 9,144 into the quadratic equation and use the quadratic formula to solve for *t*.

   $$y = -4.9t^2 + 87.21t + 9144$$

   $$9144 = -4.9^2 + 87.21t + 9144$$

   $$0 = -4.9t^2 + 87.21t$$

   $$0 = t(-4.9t + 87.21)$$

   $$t = 0\,\text{s}, 17.8\,\text{s}$$

   The plane starts the parabolic maneuver at 0 seconds and ends at 17.8 seconds, thus the astronaut feels 17.8 seconds of weightlessness on each parabola.

   To use the graphing calculator to help show the student what this looks like, graph the function $y = -4.9t^2 + 87.21t + 9144$ in y$_1$ and 9144 in y$_2$. Adjust the viewing window and graph.


   ```
   WINDOW
    Xmin=0
    Xmax=25
    Xscl=2
    Ymin=8500
    Ymax=10000
    Yscl=100
    Xres=1
   ```
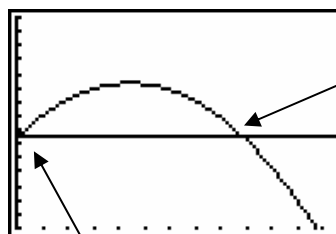
   At 9144 m (-45°, *t* = 17.8 s), parabolic motion stops.

   At 9144 m (45°, *t* = 0 s), parabolic motion starts.

3. Find the maximum altitude of the plane during one parabolic maneuver.

   Using the roots found in #2, the axis of symmetry would be x = 8.9, thus the plane reaches its maximum altitude at 8.9 seconds. Substituting this value in to the equation gives:

$$y = -4.9(8.9)^2 + 87.21(8.9) + 9144$$

$$y = 9532.0\,\text{m}$$

4. What percent of the astronaut's total flight was spent in microgravity?

    The trip lasted for 2 hours which is 7,200 seconds. Each parabola lasted for 17.8 seconds and there were 50 parabolas flown.

    $$\frac{17.8 \times 50}{7200} \times 100 = 12.4\%$$

5. How many parabolas would the pilot need to complete for the astronaut to have experienced at least 15% of his flight in microgravity?

    15% of 7200 seconds is 1080 seconds.

    1080 s / 17.8 s = 60.7

    Thus the pilot would need to complete 61 parabolas.

Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

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Exploring Space through Algebra
Weightless Wonder – Reduced Gravity Flight

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Fax the completed form to: (281) 461-9350 – Attention: Monica Trevathan

Or type your responses in an email and send to: Monica.Trevathan-1@nasa.gov

Please circle the appropriate response.

1. This problem was useful in my classroom.   YES   NO

2. The problem successfully accomplished the stated Instructional Objectives.   YES   NO

3. I will use this problem again.   YES   NO

4. Please provide suggestions for improvement of this problem and associated material:

   _____

   _____

   _____

5. Please provide suggestions for future Algebra problems, based on NASA topics, that you would like to see developed:

   _____

   _____

   _____

Thank you for your participation.

Please fax this completed form to Monica Trevathan at (281) 461-9350.